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Title: Just-non-SRI*-groups

Creator: HDML

BULLETIN OF THE
GREEK MATHEMATICAL SOCIETY
VOLUME 42, 1999 (33 – 42)

Just-non-SRI^*-groups

by
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ABSTRACT.

In the present paper we study groups, all proper factor - groups of which are generalized supersoluble. Our aim is the study of just - non- hypercyclic groups, but some results are true for the wider class of just - non - SRI^* - groups. In particular, for such a group G the structure of the Fitting subgroup $FittG$ and that of the factor group $G/FittG$ are investigated. [1] [2]

1. Introduction.

The influence of the proper quotients of a given group is one of the most important tools in studying several problems concerning the structure of infinite groups. For example, the finite factor - groups play a very important role in algorithmic problems in finitely presented groups (A.I. Maltsev, [8]) or in varieties of groups (H. Neumann, [9]). The important Robinson's Theorem ([10], Vol.2, Theorem 10. 51) on the nilpotency of finitely generated soluble groups with all finite factor - groups nilpotent, and its generalization are examples in this direction. Also it is known that the factor groups by the centralizers of chief factors play an important role in formation theory, and allow us to consider locally defined formations (see [2, Chapter IV]).

The broadest family of factor - groups is the family of all proper factor - groups. If H is a non - identity normal subgroup of a group G, then G/H is called a proper factor - group. Let X be a class of groups. A group G is said to be a just - non - X - group, if G is a non - X - group but all proper factor - groups of G are X - groups. The structure of just - non - X - groups has already been studied for several choices of the class X. In particular, D.J.S. Robinson, J.S. Wilson [11] considered soluble groups, whose proper factor - groups are supersoluble. In this connection it is natural to consider groups, all proper factor - groups which are generalized supersoluble. For instance, one could consider groups with supersoluble conjugacy classes and hypercyclic groups. Our aim is the study of groups, all proper factor-groups which lie in these classes. However some theorems are true for a wider class.

A group G is called an SRI^* - group, if G has an ascending series of normal subgroups , every factor of which is locally cyclic (S.N. Chernikov [1]). The class of SRI^* - groups includes the class of groups with supersoluble conjugate classes and

[1]1991 Mathematics subject classification 20F19,20F14

[2]This work supported by University of Athens (Research center,Grant No. 3073)

the class of hypercyclic groups. In the present paper we consider some properties of a just - non - SRI^* - groups. If G is a simple group then G is just- non- SRI^* - group for every class groups X. This shows that we need some natural restriction on a group G. The typical restriction in such investigation is $FittG \neq <1>$. In other words, a group G must contain the non - trivial normal abelian subgroup. The first main result of this paper describes a Fitting subgroup of a just - non - SRI^* - groups.

THEOREM 1 *Let G be a just - non - SRI^* - group. If $FittG \neq <1>$ then either $FittG$ is a torsion - free abelian subgroup or a elementary abelian p - subgroup for some prime p.*

The other theorems describe the structure of periodic subgroups of Fitting factor - group $G/FittG$.

Let G be a group, $P(G)$ a maximal normal periodic subgroup of G. A subgroup $P(G)$ is called the periodic part of a group G.

If H is a group then $SocH$ is a subgroup generated by all minimal normal subgroups of H. Clearly, $SocH$ is a direct product of some minimal subgroup of H. If H is an SRI^*-group the $SocH$ is a direct product of G - invariant subgroups of prime order. In particular, $SocH \leq FC(H)$.

THEOREM 2 *Let G be an infinite just - non -SRI^* - group and suppose' that $A = FittG$ is a non-identity elementary abelian p-subgroup for some prime p. Let $H = G/A$. Then*

(1).- $O_p(H) = <1>$.

(2).- If A is not a minimal normal subgroup of G, then $P(H)$ is finite.

(3).- If A is a minimal normal subgroup of G (thus, A is a monolith of G) then $SocH$ is a p' - subgroup including a subgroup J such that $SocH/J$ is locally cyclic and $Core_H(J) = <1>$.

Let G be a group, A normal abelian subgroup of G. We say that A is rationally irreducible if for every non - identity G - invariant subgroup B of A a factor - group A/B is periodic.

THEOREM 3 *Let G be an infinite just - non - SRI^* - group and suppose that $A = FittG$ is a torsion - free subgroup. Let $H = G/A$. Then*

(1).- If A is not rationally irreducible then $P(H)$ is finite.

(2).- If A is rationally irreducible, then $SocH$ contains a subgroup J such that $SocH/J$ is locally cyclic and $Core_H(J) = <1>$.

2. The structure of the Fitting subgroup

The aim of this section is to prove that any just - non-SRI^*-group with non-trivial Fitting subgroup has a Fitting subgroup, a torsion- free abelian subgroup or an elementary abelian p - subgroup for some prime p.

LEMMA 1 *Let G be a just - non - SRI^* - group. Then G does not contain the non - trivial normal subgroups H_1, H_2 such that $H_1 \cap H_2 =< 1 >$.*

PROOF: Suppose the contrary . By Remak's theorem we obtain the embedding $G \leq G/H_1 \times G/H_2$. It follows that G is an SRI^* - group, because G/H_1 and G/H_2 are SRI^* - groups. □

COROLLARY 1 *Let G be a just - non - SRI^* - group. If G includes a finite normal subgroup then G is finite.*

PROOF: Suppose that G is infinite. Let A be a finite minimal normal subgroup of G. Then either A is a normal elementary abelian p - subgroup for some prime p, or A is a non - abelian semisimple subgroup. In the second case $A \cap C_G(A) =< 1 >$. Since A is finite then $G/(C_G(A))$ is finite, in particular, $C_G(A) \neq< 1 >$. We obtain the contradiction with Lemma 1. Hence A is an elementary abelian p - subgroup. Since $G/C_G(A)$ is a finite SRI^* - group then it is supersoluble. By Theorem 1 of paper [12] G is a semidirect product of A and H where $H \cong G/A$ is an SRI^* - subgroup. Since G is infinite then H is infinite too. Moreover, $C_H(A)$ is an infinite normal subgroup of H. Then $C_H(A)$ includes a non - trivial H - invariant locally cyclic subgroup L. Since $G = AH$ then L is normal in G. Since G/L is an SRI^* - group, then G is an SRI^* - group too. This is a contradiction. Thus G is finite. □

LEMMA 2 *Let G be a just - non - SRI^* -group, A a non-identity normal abelian subgroup of G. Then either A is an elementary abelian p - subgroup for some prime p, or A is torsion - free.*

PROOF: Let T be the torsion part of A. Suppose that $T \neq< 1 >$. Lemma 1. yields that T is a p - subgroup for some prime p. Put $T_1 = \Omega_1(T) = \{x \in T | x^p = 1\}$. Suppose that $T_1 \neq T$, then $T_1 \neq T_2 = \Omega_2(T) = \{x \in T | x^{p^2} = 1\}$. Consider the mapping $\phi : x \rightarrow x^p$, $x \in T_2$. Clearly ϕ is a $\mathbf{ZG}$ - endomorphism of T_2. Since G/T_1 is an SRI^* - group then T_2/T_1 includes a non - identity G - invariant cyclic subgroup C/T_1. Since $C \neq T_1$, then $C\phi \neq< 1 >$. So $C\phi \cong C/(C \cap ker\phi) = C/T_1$. Hence $C\phi$ is a non-identity normal cyclic subgroup of G. This means that G is an SRI^* - group. This contradiction shows that $T = \Omega_1(T)$. In other words, T is an elementary abelian p - subgroup. In this case $A = T \times U$ for some subgroup U (see, for example, [5, Theorem 27.5]). If we assume that $A \neq T$ then it follows that $< 1 > \neq A^p \leq U$, in particular $A^p \cap T =< 1 >$. This contradicts Lemma 1. Hence $A = T$, that is A, an elementary abelian. □

PROPOSITION 1 *Let G be a just - non - SRI^* - group, L a normal nilpotent subgroup of G. Then L is abelian.*

PROOF: **case 1**. If L is a normal torsion - free nilpotent subgroup of G. Then, suppose that L is non - abelian. Let A be a maximal G - invariant abelian subgroup of L. Then $A \neq L$. Let T/A be the torsion part of L/A. If $a, b \in T$ then $a^k, b^l \in A$ for some $k, l \in \mathbf{N}$. In particular, $a^k b^l = b^l a^k$. It follows that $ab = ba$ (see, for example [7, §66]), that is T abelian. From the choice of A we obtain that $A = T$, in particular, L/A is torsion - free. Since $A \neq < 1 >$ then G/A is an SRI^* - group. Let $Z/A = \zeta(L/A)$, then Z/A is a non - identity G - invariant subgroup. Therefore Z/A includes a G - invariant locally cyclic subgroup X/A. It follows from the choice of A that X is non - abelian.
Let $Z_1 = \zeta(L)$, then $Z_1 \neq < 1 >$ and Z_1 is G - invariant subgroup of L. From the choice of A we obtain that $Z_1 \leq A$. Moreover, $Z_1 \leq C_G(X)$, in particular, $C_A(X)$ is a non - identity G - invariant subgroup of A.
First assume that $A \neq C_A(X)$. Then $G/C_A(X)$ is an SRI^* - group, thus $A/C_A(X)$ includes a non - identity G - invariant locally cyclic subgroup $B/C_A(X)$. The factor - group $L/C_A(X)$ is torsion - free (see, for example, [7, §66]), so $B/C_A(X)$ is torsion-free too.
Since X/A is locally cyclic then there are elements x_n ($n \in \mathbf{N}$) such that $< x_1A > \leq < x_2A > \leq \ldots \leq < x_nA > \leq \ldots$ and $X/A = \cup_{n \in \mathbf{N}} < x_nA >$. Since $C_B(x_nA)$ is a pure subgroup and $B/C_A(X)$ is locally cyclic then either $B = C_B(x_n)$ or $C_B(x_n) = C_A(X)$. Since $C_B(x_n) \geq C_B(x_{n+1})$ then there is a number m such that $C_B(x_m) = C_A(X)$. We can assume that $m = 1$.
Let $x \in X$ and consider the mapping $\phi_x : b \rightarrow [b, x], b \in B$. We have

$$(bc)\phi_x = [bc, x] = [b, x]^c[c, x] = [b, x][c, x] = b\phi_x.c\phi_x$$

that is ϕ_x is an endomorphism of B. Further, $Im\phi_x = [B, x]$ and $ker\phi_x = C_B(x)$, so that $[B, x] = Im\phi_x \cong B/ker\phi_x = B/C_B(x)$. In particular, $[B, x_n]$ is a locally cyclic subgroup for any $n \in \mathbf{N}$. Since $[B, x_n]$ is a subgroup, then $[B, x_n] = [B, < x_n >]$. It follows that

$$[B, x_n] = [B, < x_n >] \leq [B, < x_{n+1} >] = [B, x_{n+1}].$$

Therefore $[B, X]$ is a join of ascending chain

$$[B, x_1] \leq [B, x_2] \leq \ldots \leq [B, x_n] \leq \ldots$$

of locally cyclic subgroups. This means that $[B, X]$ is locally cyclic too, since $B \neq C_B(X)$ then $[B, X] \neq < 1 >$; and since B and X are normal subgroups then $[B, X]$ is normal too. Hence G includes a normal locally cyclic subgroup. It follows that G is an SRI^* - group. Contradiction.

Now consider the case when $A = C_A(X)$. In other words, $A \leq \zeta(X)$. Since X/A is locally cyclic, then X is abelian, contradicting the choice of A. This contradiction shows that A is abelian.

case 2. If L is a normal torsion nilpotent subgroup of G. Then, again suppose that L is non - abelian. Let A be a maximal G - invariant abelian subgroup of L. Since L is nilpotent, then $A \neq < 1 >$. So G/A is an SRI^* - group. It follows that L/A includes a G - invariant non-identity locally cyclic subgroup. Since L is torsion, then this means that L/A includes a G - invariant non - identity cyclic subgroup $X/A = < xA >$. First consider the case when A is not a subgroup of $\zeta(X)$. In other words $C_A(X) \neq A$. Since L is nilpotent, then $\zeta(L) \cap A \neq < 1 >$. In particular, $C_A(X) \neq < 1 >$. Hence $G/C_A(X)$ is an SRI^* - group, so $A/C_A(X)$ includes a G - invariant non - identity locally cyclic subgroup $B/C_A(X)$. Again consider the mapping $\phi_x : b \rightarrow [b, x]$, $b \in B$.

This mapping is an endomorphism of A, $Im\phi_x = [B, x]$, $ker\phi_x = C_B(x)$. Thus $[B, x] = Im\phi_x \cong B/ker\phi_x = B/C_B(x)$. Since $X/A = < xA >$ then $[B, x] = [B, X]$ and $C_B(x) = C_B(X)$. It follows that $[B, X]$ is cyclic. Since B and X are normal subgroups then $[B, X]$ is normal too. Since $B \neq C_A(X)$ then $[B, X] \neq < 1 >$. It follows that G is an SRI^* - group. Contradiction.

Assume now that $A \leq \zeta(X)$. Since X/A is cyclic then X is abelian . However, this contradicts to the choice of A.

Let T be the torsion part of L. If $L = T$ then *case* 2 yields that L is abelian. It $T = < 1 >$ then *case* 1 yields that L is a abelian. Therefore we must consider the case when $< 1 > \neq T \neq L$. *Case* 1 yields that T is abelian. Lemma 2 implies that T is elementary abelian p - subgroup for some prime p. From Proposition 2 of paper [6] we obtain that L includes a normal torsion - free subgroup V such that L/V is bounded. In other words, there is a number $t \in \mathbf{N}$ such that $L^t \leq V$. Since L is not torsion then $L^t \neq < 1 >$, and since $L^t \leq V$, then L^t is torsion - free. Hence G includes two non - identity normal subgroups L^t and T such that $L^t \cap T = < 1 >$. Contradiction of Lemma 1. □

PROOF OF THEOREM 1 : Let $x, y \in FittG$, then there exists the normal nilpotent subgroups L_x, L_y such that $x \in L_x$, $y \in L_y$. By Fitting Theorem (see, for example, [10, Theorem 2.18]) L_xL_y is normal nilpotent subgroup. By Proposition 1 L_xL_y is abelian, in particular, $xy = yx$ and now we must apply Lemma 2. □

COROLLARY 2 *Let G be a just - non - SRI^* - group. If $FittG = A \neq < 1 >$, then $A = C_G(A)$.*

3. The periodic subgroups of the factor - Group G/FittG.

The object of this final section is to describe the periodic subgroups of factor - group $G/FittG$. The description of such subgroups is given in Theorem 2 and Theorem 3.

LEMMA 3 *Let G be a group, E an abelian normal subgroup of G, A a G - invariant subgroup of E, $C = C_G(A)$, x an element of G such that the subgroup $< xC >$ is normal in G/C. Then the subgroups, $C_A(x)$ and $[A, x]$ are G - invariant in E.*

PROOF: Indeed, let a be an element of $C_A(x)$ and g an element of G. Then $gxg^{-1} = x^n c$ for some $c \in C$, so that $gx = x^n cg$ and $(gx)^{-1} = g^{-1}c^{-1}x^{-n}$. We have now $(a^g)^x = x^{-1}(g^{-1}ag)x = (gx)^{-1}a(gx) = g^{-1}c^{-1}x^{-n}ax^n cg = g^{-1}ag = a^g$. It shows that $C_A(x)$ is a G - invariant subgroup. Consider now the element $[a, x^2]$ for each $a \in A$. Then $[a, x^2] = [a, x][a, x]^x = [a, x][a^x, x^x] = [a, x][a^x, x] \in [A, x]$. Assume that we have already proved that $[a, x^k] \in [A, x]$. Then $[a, x^{k+1}] = [a, xx^k] = [a, x^k][a, x]^{x^k} = [a, x^k][a^{x^k}, x] \in [A, x]$. Let $g \in G$, then $x^g = g^{-1}xg = x^n c_1$ for some element $c_1 \in C$. Now we have for each $a \in A$, $[a, x]^g = [a^g, x^g] = [a^g, x^n c_1] = [a^g, c_1][a^g, x^n]^{c_1} = [a^g, x^n] \in [A, x]$. □

PROPOSITION 2 *Let G be a just - non - SRI^* - group , $A = FittG$, B a non - identity G - invariant subgroup of A , $C = C_G(B)$. If x is an element of G/C such that $< xC >$ is normal in G/C then $C_B(x) =< 1 >$.*

PROOF: Assume the contrary, and let $B_1 = C_B(x) \neq< 1 >$. Since G/B_1 is an SRI^* - group then B/B_1 includes a non - identity locally cyclic G - invariant subgroup B_2/B_1. Then $C_{B_2}(x) = C_B(x)$ and $[B_2, x]$ are G - invariant subgroup of A by Lemma 3. Moreover, the mapping $\phi : b \to [b, x]$, $b \in B_2$ is a homomorphism of B_2 onto $[B_2, x]$ with $Ker\phi = C_{B_2}(x) = B_1$. Then $[B_2, x] = Im\phi \simeq B_2/B_1$ is a non - identity locally cyclic normal subgroup of G. It follows that G is an SRI^* - group. This contradiction shows that $C_B(x) =< 1 >$. □

COROLLARY 3 *Let G be an infinite just - non - SRI^* - group, $A = FittG$, B a non - identity G - invariant subgroup of A. Suppose that A is elementary abelian p - subgroup for some prime p. Then $O_p(G/C_G(B)) =< 1 >$.*

PROOF: Corollary 1 yields that B is infinite. Let $C = C_G(B)$. Assume that $O_p(G/C) \neq< 1 >$. Since G/C is a SRI^* - group then $O_p(G/C)$ includes a non - identity G - invariant cyclic p - subgroup $< xC >$. Then $x^{p^k} \in G$ for some $k \in \mathbf{N}$. Let $1 \neq b \in B$, $B_1 =< b >^{<x>}$. Then B_1 is finite elementary abelian p - subgroup and $x^{p^k} \in C_G(B_1)$. Let $F =< b, x >=< B, x >$. A factor - group $F/\zeta(F)$, is a p - group, because x^{p^k} belongs to $\zeta(F)$, therefore a subgroup F is nilpotent. Since $F \cap B$ is a non - identity normal subgroup of F, then $\zeta(F) \cap (F \cap B) \neq < 1 >$. It follows that $C_B(x) \neq < 1 >$. Contradiction of Proposition 2. □

PROPOSITION 3 *Let G be an infinite just - non - SRI^* - group, $A = FittG$. Suppose that A is a non - trivial elementary abelian p-subgroup for some prime p. If U/V is a G - factor of A, then $P(C_G(U/V)/C_G(U)) =< 1 >$.*

PROOF: Suppose the contrary. Then $C_G(U/V)/C_G(U)$ includes a non - trivial G-invariant cyclic q-subgroup $< xC_G(U) >$ where q is prime. By Corollary 1 the subgroup U is infinite and a just-non-SRI^*-group. Let $C = C_G(U)$, $\bar{x} = xC$. By Corollary 3 $p \neq q$. We can consider $< \bar{x} >$ as finite q - group of automorphisms of the elementary abelian p - group U. By Maschke's Theorem there exists a $< \bar{x} >$ - invariant subgroup W such that $U = V \times W$. If $a \in W$, then $[a, x] \in W$, because W is a $< \bar{x} >$ - invariant. It follows that $[a, x] \in W \cap V =< 1 >$. In other words, $W \leq C_U(x)$, and we obtain the contradiction with Proposition 2. □

COROLLARY 4 *Let G be a just - non - SRI^* - group, $A = FittG$. Suppose that A is a non - trivial elementary abelian p-subgroup for some prime p.*
(1).-Let G be non - monolithic, then $P(G/A)$ is finite.
(2).-Let G be a monolithic group with a monolith M. If $M \neq A$ then $P(G/A)$ is finite.

PROOF: Corollary 1 yields that a non - monolithic just - non - SRI^* - group is infinite. By Corollary 2 , $C_G(A) = A$. Suppose that the subgroup $P/A = P(G/A)$ is infinite. Since G is a non - monolithic group then A includes a G - invariant cyclic factor E/B. Put $C = C_G(E/B)$. Since E/B is finite, then the G/C is finite too. In particular, $T/A = (P \cap C)/A$ is infinite. By Proposition 3 the factor - group $C/C_G(E)$ has a trivial periodic part. It follows that $T \leq C_G(E)$. Since G/A is an SRI^* - group then T/A includes a cyclic G invariant subgroup $< xA >$. By Proposition 2 $C_A(x) =< 1 >$. But on other hand $< 1 > \neq E \leq C_A(T) \leq C_A(x)$. This contradiction shows that $P(G/A)$ is finite.
Since G/M is an SRI^* - group then A/M includes a G - invariant cyclic subgroup E/M. Using the same arguments as in the proof of Corollary 3, we obtain the contradiction. □

PROPOSITION 4 *Let G be a just - non - SRI^* - group, $A = FittG$. Suppose that A is a non - identity torsion - free subgroup. If A is not rationally irreducible then $P(G/A)$ is finite.*

PROOF: Assume that the Proposition is false. By Theorem 1 A is a torsion - free abelian subgroup. Since A is not rationally irreducible then A includes a non - identity G - invariant subgroup B such that A/B is not periodic. Let E/B be the periodic part of the abelian group A/B. Clearly E is a G - invariant subgroup of A and A/E is a non - trivial torsion - free group. Since G/E is an SRI^* - group then A/E includes a G - invariant locally cyclic subgroup U/E.
Let $P/A = P(G/A)$. By Corollary 2 $A = C_G(A)$. Suppose that P/A is infinite. Let $G = C_G(U)$, then $G \geq A$. If we assume that $P \cap C \neq A$, then $(P \cap C)/A$ includes finite cyclic subgroup $< zA >$. It follows that $U \leq C_A(z)$. But this contradicts Proposition 2. This contradiction shows that $P \cap C = A$, so that $T/C = PC/C \cong P/(P \cap C) = P/A$ is infinite.
Since U/E is a torsion - free locally cyclic group, then every periodic subgroup of Aut(U/E) is finite (see, for instance [13, Theorem 9. 23]). So $T/C_T(U/E)$ is finite.

In particular, $C_T(U/E)/C$, is infinite. Let $< xC >$ be a non - identity cyclic G - invariant subgroup of $C_T(U/E)/C$, $\bar{x} = xC$. Then we can consider $< \bar{x} >$ as a finite automorphism group of a torsion - free abelian subgroup U. Let $vE \in U/E$, $V/E =< vE >^{<xE>}$. Then V/E is a finitely generated torsion - free abelian group, so that V/E is a free abelian.
In this case, the subgroup E has a complement in V (see, for example, [5, Theorem 14.4]). By generalized Maschke's Theorem [3, Theorem 2.7] V includes a $< \bar{x} >$ - invariant subgroup W such that $W \cap E =< 1 >$ and the index $|V : WE|$, is finite. In particular, $W \neq< 1 >$. If $w \in W$ then $[w, x] \in E$ because $x \in C_T(U/E)$, then $[w, x]$ belongs to $W \cap E =< 1 >$. In other words, $W \leq C_U(x)$, contradicting Proposition 2. This contradiction shows that P/A is finite. □

Let G be a just - non - SRI^* - group such that $P(G/A)$ is infinite, where $A = FittG$. Suppose that A is torsion - free. Proposition 4 yields that in this case A is rationally irreducible. In particular, if G is a monolithic group then A is its monolith of G. Since $A = C_G(A)$ then in this case we can consider A a simple **ZH** - module, where $H = G/A$ is an SRI^* - group. As A is torsion - free then $A{=}A^n$ for any $n \in \mathbf{N}$, so that A is a divisible abelian subgroup. Therefore we can consider A a simple **QH** - module.
Suppose now that G is non - monolithic. Again we can consider A a **ZH** - module. Put $E = A \otimes_{\mathbf{Z}} \mathbf{Q}$. Then we can consider E a **QG** - module , moreover $C_G(E) = C_G(A) = A$, so that E is a **QH** - module. Since A is rationally irreducible then E is a simple **QH** - module.
Finally, if A is an elementary abelian p - group for some prime p, then Corollary 4 yields that G is a monolithic group. Let M be the monolith of G. Corollary 4 shows that in this case $A = M$. So again we can consider A as a simple $\mathbf{F_pH}$ - module. Consequently, we must consider simple **FH** - modules, where F is a field, H is an SRI^* - group.
For this case we need the following Proposition. Recall here that the (FC) *– centre* $FC(G)$ of a group G is the subgroup consisting of all elements of G having finitely many conjugates.

PROPOSITION 5 *Let F be a field, G a group, A a simple* **FG** *- module such that $C_G(A) =< 1 >$. Let P be a G - invariant elementary abelian p - subgroup of $FC(G)$, p is prime. Then $charF \neq p$, and P includes a subgroup J such that $|P/J| = p$ and $Core_G(J) = < 1 >$.*

PROOF: The proof is the same as a proof of Lemma 8.2 of paper [4]. □

PROOF OF THEOREM 2 : (1) follows from Corollary 3. Assume now that A is not a minimal normal subgroup of G. If G is non - monolithic then $P(H)$ is finite by (1) of Corollary 4. If G is monolithic, then we will apply (2) of Corollary 4.

Finally consider a case when A is the monolith of G. Then we can consider A a $\mathbf{F_pH}$ - module. Let $S = SocH$, Q_r a Sylow r - subgroup of S for every prime r. By Proposition 5 $r \neq p$ and Q_r includes a subgroup J_r with the following properties: $|Q_r/J_r| = r$ and $Core_G(J) =< 1 >$. Put $J = \mathbf{X}_{r \in S} J_r$. Then S/J is a locally cyclic group and $Core_H(J) =< 1 >$. □

PROOF OF THEOREM 3 : If A is not rationally irreducible then $P(H)$ is finite by Proposition 4. Let now A be a rationally irreducible in G. We can consider A as a $\mathbf{ZH}$ - module. Put $E = A \otimes_{\mathbf{Z}} \mathbf{Q}$, then E is a simple $\mathbf{QH}$ - module and $C_H(E) =< 1 >$. Moreover, if G is monolithic, then A is divisible and therefore $E = A$. In this case we can apply Proposition 5. □

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